How can the volume & surface area of a chicken egg be calculated?

SEPEHR AKBARI

MATHEMATICAL INVESTIGATION
INTERNAL ASSESSMENT

DATE: 10/2/22

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1. Introduction

1.1 Motivation

From the first days when I was introduced to basic theories of mathematics, physics, and even computer science, I have been intrigued to look at real-life applications around me, through those lenses. As I come from an education system that doesn't view real-life applications and is more focused on theories, the only place I could explore applications was outside classes and in my daily life. As I was always interested and excited about these sciences, I unintentionally began looking at everything in my routine life from a problem-solving perspective. For example, a couple of weeks ago, a friend and I decided to calculate the approximate slope of my city using the measures of each individual pole of a bridge. These kinds of investigations are extremely interesting to me, as they give me a chance to apply different aspects of my knowledge to reach a certain answer or conclusion. Therefore for this mathematical investigation, I have decided to employ my mathematical knowledge to something of my interest, from my routine life, and investigate it in depth, as much as my knowledge allows me to.

While searching for an interesting topic, I encountered a chicken egg that I left on the kitchen table roll and fell. Although it seems unreal, that was the first time, I actually started looking at the shape of an egg, an object I see every day. Therefore I've decided to explore the chicken egg, by modeling through different perspectives and approaches.

1.2 Aim & Approach

This investigation aims to find an appropriate mathematical model for a chicken egg and compare them. Furthermore, I've decided to calculate the amount of yoke within the egg which indicates the volume of the egg, and the amount of eggshell on a chicken egg, which is in other words the surface area of the egg.

To start the investigation, I picked up an egg from my fridge and took a picture of it. This is done so I can get the egg's dimensions, create functions for them, and model them. Moreover, I can compare the results of my modelings with the actual object, to prevent any errors from happening. I will be using three different methods to model the chicken egg, and after doing so I will try to evaluate which one is more precise, taking different aspects into account.

2. Modeling the Egg

To correctly model the egg and have maximum accuracy in measuring the volume and surface area, a dimension of the egg needed to be measured. I decided to measure the length of the egg, and to do so I needed to cook the egg, cut the egg in half, and measure the length. However, the egg must be cut in half on its exact axis of symmetry to assure precision. To find the axis of symmetry of the egg, I drew 3 lines on the egg, one of which was where I thought the axis of symmetry approximately is, and the other two, were about 2mm away from the first line from each side. I then took a picture of the egg and used a Wolfram Algorithm ***(Appendix A)*** to find the exact axis of symmetry (Nicki Estner). As I had the lines on the egg, I knew exactly where the axis of symmetry lays, and therefore drew it on the egg with their help, and cut the egg accordingly. I then used a ruler to measure the length, which came up to be exactly 5.5cm. It is important to acknowledge that however the axis of symmetry was found with maximum precision, it was drawn on the egg by hand, which could involve errors. Moreover, the precision of the ruler I used to measure the length is 0.1cm, which results in an uncertainty of ±0.1cm in the measurement.

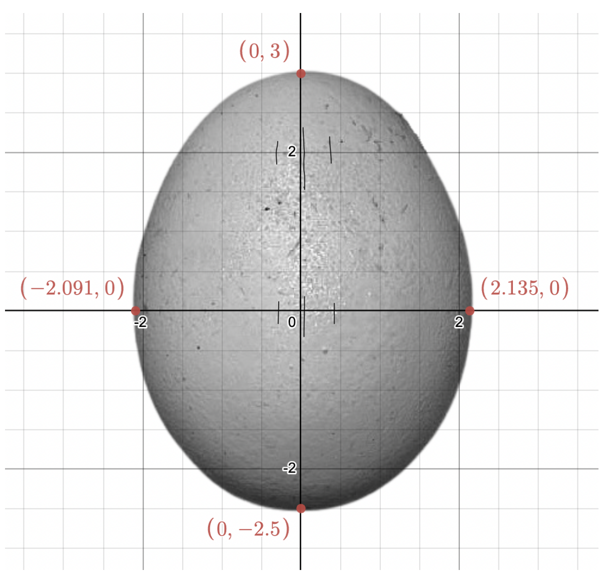

Figure 2.1: Cropped version of the the egg in desmos, showing a length of 5.5cm.

2.1 Ellipse Equation

, Although the shape of an egg differs from an ellipse, by modifying the ellipse equation the shape of an egg can be structured as well. As the ellipse equation is already explored and has an exact formula, it is a good starting point for this investigation. The equation of the general ellipse is:

$$[1] \qquad \frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

The major and minor axes in the equation above, are each called "radii". $\pm r_x^2$ and $\pm r_y^2$ represent the horizontal and vertical radii, or the x and y axes-intercepts. The distance between the two vertical horizontal and vertical intercepts is called magnitude, which is twice the value or r_x or r_y. The one axis with the greater magnitude is known as the major axis and the other one is the minor axis.

As I aim to resemble the shape of the egg, I need to change the form of the general ellipse equation into a function, which can be simply done by adding the term 'cy' to the denominator.

$$[2] \qquad \frac{x^2}{r_x^2} + \frac{y^2}{r_y^2 + cy} = 1, c > 0$$

The term was added as 'c' now has the ability to control the shape above and under the x-axis, and modify the shape of the ellipse in order to make it into an egg. To explain how this works, we need to look at the symmetry of the ellipse. The denominator of the equation was r_y^2, which is a constant, therefore giving the same vertical radius on both sides of the x-axis. In order to change the symmetry and alter the shape, I added another denominator, which is dependent on y. It will go below or above the x-axis depending on the change of magnitude. When the value of 'y' is positive, the magnitude of '$r_y^2 + cy$' will increase (while $c > 0$). Vice versa, the magnitude would decrease when 'y' is negative. A change in magnitude means that the distance from the axis will change, therefore this asymmetry will result in a change of distance from the origin points on the curve. And so as ‘c’ gets larger in value, the asymmetry gets more drastic. Below is the comparison of each equation, which can prove the higher accuracy of the function:

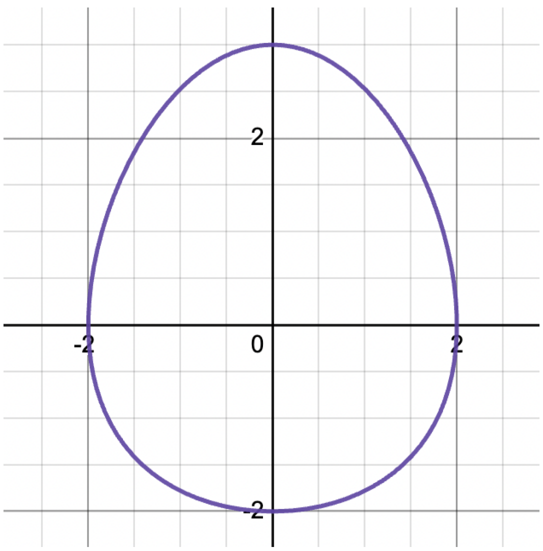

Figure 2.2: result of equation [2]

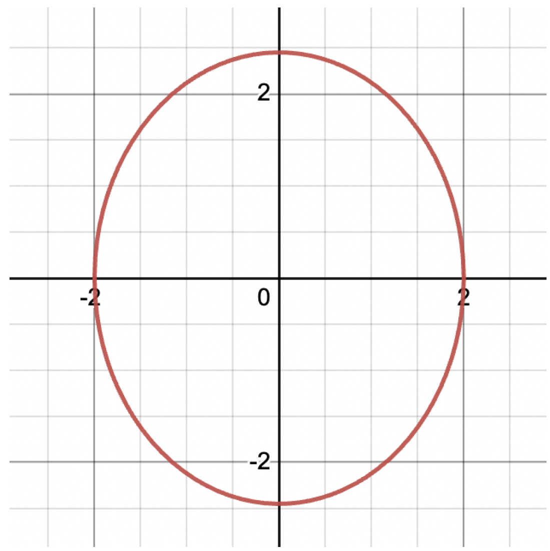

Figure 2.3: result of equation [1]

Note: The numbers on the graphs above are completely hypothetical, and they are used for graphing purposes only.

To apply this model to our egg, we need to find appropriate answers for r_x^2, r_y^2 and c. This can be done by making a system of three linear equations requiring three coordinates. Looking back at **Figure 2.1**, we can see the x-axis's interception at coordinates $(\pm 2.1, 0)$, and the y-axis's at coordinates (0,3) and $((0, -2.5)$. Using this information and substituting them in equation [2] will give us these three linear equations:

$$[3] \qquad \frac{x^2}{r_x^2} = 1 \rightarrow \frac{(2.1)^2}{r_x^2} = 1$$

$$[4] \qquad r_y^2 + c\,y \rightarrow r_y^2 + 3c = 3^2\,, \quad [5] \qquad r_y^2 + c\,y \rightarrow r_y^2 - (2.5)c = (2.5)^2$$

To solve for r_x^2 in [3], we should simply calculate $(2.1)^2$ as $r_x^2 = (2.1)^2$. Therefore $r_x^2 = 4.4$. Moreover, to solve 'c' we can subtract [5] from [4]:

$$(r_y^2 + 3c) - (r_y^2 - 2.5c) = 9 - 6.26$$

$$\rightarrow 5.5c = 2.75 \rightarrow c = \frac{2.75}{5.5} = 0.5$$

Knowing $c = 0.5$, we can substitute it in [4] or [5] and solve for r_y^2:

$$r_y^2 + 3(0.5) = 9$$

$$\rightarrow r_y^2 = 9 - 1.5 \rightarrow r_y^2 = 7.5$$

According to the value of each variable, we can now form the equation of our specific egg:

$$[6] \qquad \frac{x^2}{4.4} + \frac{y^2}{7.5 + (0.5)y} = 1$$

With the above explanations, I expect the equation to pass all four axes' intercepts and have somewhat similar curvatures to the egg, showing a general model of the egg.

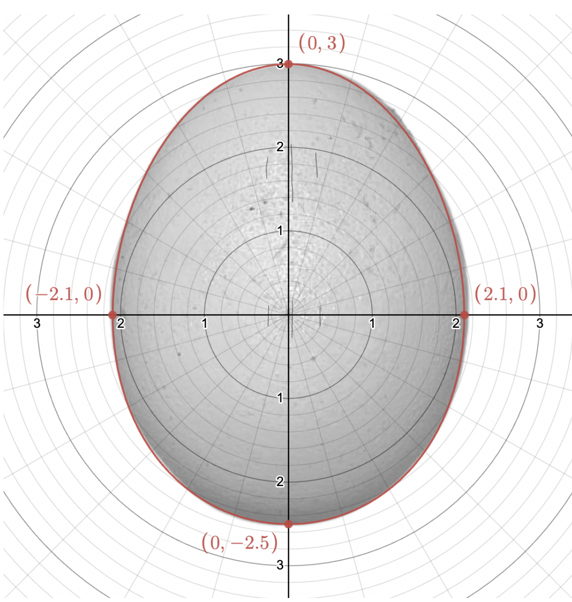

Figure 2.4: Curve of [6] in comparison to the egg

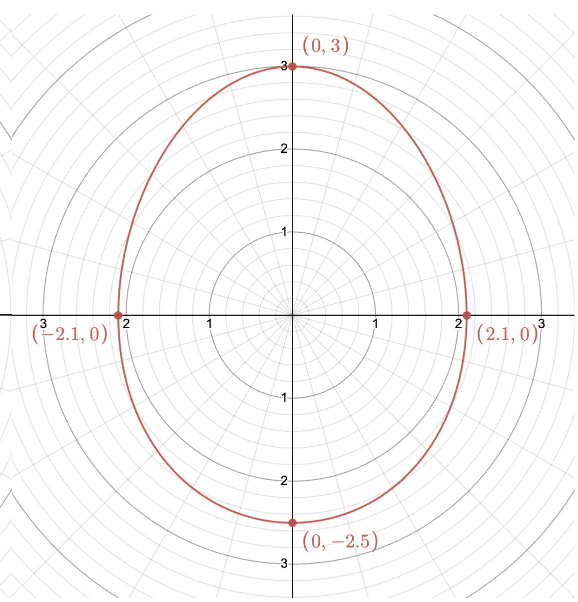

Figure 2.5: The curve produced from [6]

Although the equation [6] successfully models the egg shape and passes the expectations mentioned above, in closer observation, the curve seems to minimally extend beyond the egg's curvature below the x-axis. Although the shape would've been more accurate if different coordinates on the egg were used to calculate the coefficients, it would've still not been perfectly aligned with the egg as the modified Ellipse equation is not perfectly accurate. To make this equation more accurate, perhaps another term should've been put into [1] rather than 'cy', composing a different function that could potentially raise accuracy. However this matter is beyond the aim of this investigation, and the result above is acceptable for me, as it is only slightly different from what could be considered "a perfect model".

2.2 Oviform Curve

Another way to approach modeling the shape of the egg is through the geometric formula of spheres and ellipsoids. I chose to take this approach as the curves of the equation include the shape of an ovoid which is very similar to one of an egg. The equation formula for an oviform curve (Juergen Koeller) is:

$$[7] \qquad y = \pm \frac{B}{2} \cdot \sqrt{\frac{L^2 - 4x^2}{L^2 + 8wx + 4w^2}}$$

In [7], 'L' is the egg's length which in this case is 5.5cm. 'B' represents the maximum breadth of the egg, which is in this case $2.1 + 2.1 = 4.2$cm. In the formula, 'w' represents the curves the model has, therefore $w = 0$ will result in an ellipse with equal curves. To graph [7] I used Python with matplotlib ***(Appendix B)***, and therefore got this output for [7] with $L = 5.5$, $B = 4.4$, $w = 0$:

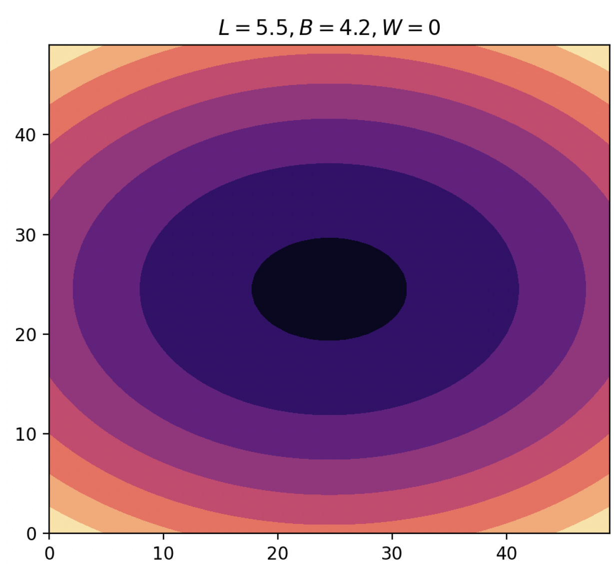

Figure 2.6: Output of Python code in Appendix B.1

Now to model the egg correctly we have to give some value to 'w' to create an extended curve that can model the egg precisely. The more the value of 'w' is, the more the curves will extend, or for a more clear understanding, the more the curves will become pear-shaped. However, it is important that the value of 'w' has a limit which is $\frac{L-B}{2}$:

$$[8] \qquad y = \pm\frac{4.2}{2}\cdot\sqrt{\frac{(5.5)^2-4x^2}{(5.5)^2+8wx+4w^2}},$$

$$[9] \qquad \rightarrow \frac{L-B}{2} \geq w \ \rightarrow 0.65 \geq w$$

To understand the role of 'w' better, which is the key element of my modeling in this approach, I wrote another Python program ***(Appendix B)*** to get a comparison between $w = 0$ and $w = 0.65$:

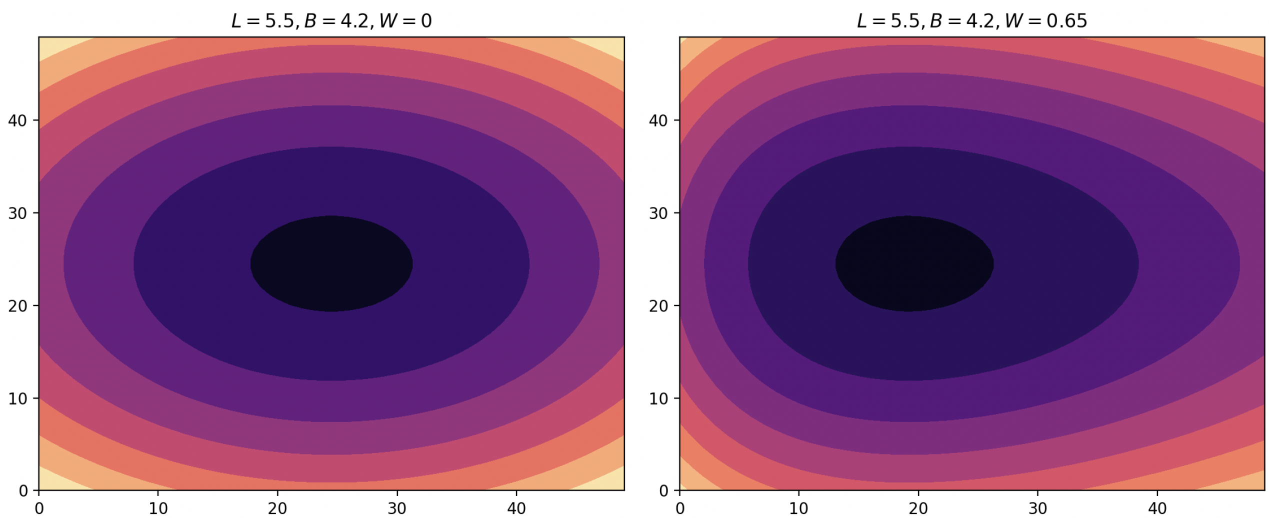

Figure 2.7: Comparison between w=0 and maximum value of w

To find the best value of 'w' which can most accurately model the egg, I tried to compare [8] with the egg, and change the value of 'w' according to [9] until it gives an output that is acceptable as a model of the egg using Desmos. After a few tests, I found the closest value of 'w' to model the egg correctly to be $w = 0.25$. This is the comparison of the maximum value of 'w' and the most accurate value of 'w' to model the egg (0.25), which clearly highlights the importance of the precision of the curve.

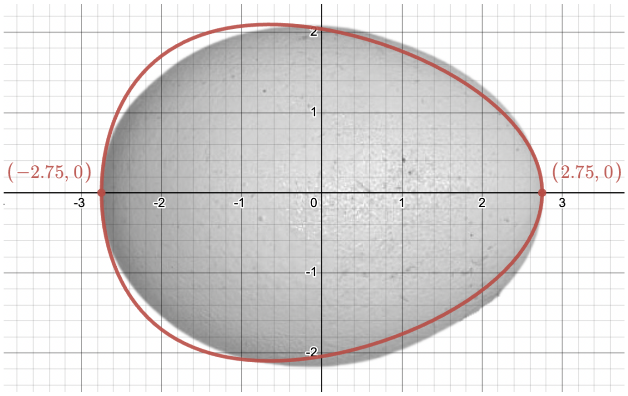

Figure 2.8: curve with w = 0.65

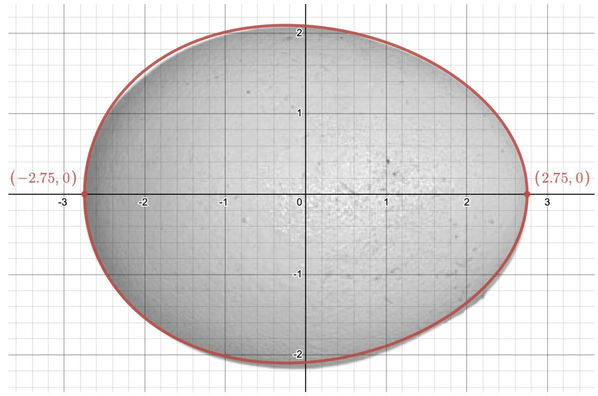

Figure 2.9: curve with w = 0.25

Figure 2.9 shows nearly perfect modeling of the egg, using the oviform curve formula. Although the modeling is still not perfect, it is acceptable as it presents the general form of the chicken egg.

2.3 Lagrange Interpolation

In this method, I will try to model the egg by using polynomials. As the shape of an egg is symmetrical, I can use polynomials to model one half of the egg and reflect the equation in the x-axis to find the equation for the other half. I decided to use the Lagrange interpolation to find the polynomials' functions which will be used for modeling the egg. The formula states that to find a unique polynomial with n degrees, $(n+1)$ points are required to find the best fit for it. The formula justified for the polynomial T(x) is:

$$[10] \qquad T(x) = \sum_{i=1}^{n+1} \frac{(x-x_1)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_{n+1})}{(x_i-x_1)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_{n+1})} y_i$$

$$\text{while } 1 \le i \le n+1,\ T(x_i) = y_i$$

The higher the degree of n, the more accurate the modeling will be. However, I will not examine the case of using high degrees such as five or six, to minimize complexity. It is important to understand that the egg's curvature's nature is not quadratic, cubic, or quartic either, therefore I will use multiple simpler polynomials to model the egg. Subsequently, I will divide the shape into three sections, the middle, and the left and right, and model them using three ($i = 3$) different quadratic polynomials ($n = 2$), therefore [10] can be written as:

$$[11] \qquad P(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}y_3$$

In order to find the appropriate function to graph a parabola for each section, I found the three points by placing the egg's picture in Desmos. I then input the coordinates in [11] and found the function for each side in the form of $ax^2 + bx + c$. The coordinates I am using are:

i	x	y
1	0	0
2	0.26	0.96
3	0.526	1.3

Table 01: Left side coordinates

i	x	y
1	0.75	1.5
2	2.5	2.12
3	4.58	1.5

Table 02: Middle side coordinates

i	x	y
1	5.5	0
2	4.85	1.3
3	5.17	0.96

Table 03: Right side coordinates

Therefore the appropriate function for the left section is:

$$P_{left}(x) = \frac{(x-0.26)(x-0.526)}{(0-0.26)(0-0.526)}0 + \frac{(x-0)(x-0.526)}{(0.26-0)(0.26-0.526)}0.96 + \frac{(x-0)(x-0.26)}{(0.526-0)(0.526-0.26)}1.3$$

$$[12] \qquad \rightarrow P_{left}(x) = \frac{-2087000x^2 + 2221612x}{454727}$$

Furthermore, the function for the middle side is:

$$P_{middle}(x) = \frac{(x-2.5)(x-4.58)}{(0.75-2.5)(0.75-4.58)}1.5 + \frac{(x-0.75)(x-4.58)}{(2.5-0.75)(2.5-4.58)}2.12 + \frac{(x-0.75)(x-2.5)}{(4.58-0.75)(4.58-2.5)}1.5$$

$$[13] \qquad \rightarrow P_{middle}(x) = \frac{-6200x^2 + 33046x + 33303}{36400}$$

Lastly, the function for the right side is:

$$P_{right}(x) = \frac{(x-4.85)(x-5.17)}{(5.5-4.85)(5.5-5.17)}0 + \frac{(x-5.5)(x-5.17)}{(4.85-5.5)(4.85-5.17)}1.3 + \frac{(x-5.5)(x-4.85)}{(5.17-5.5)(5.17-4.85)}0.96$$

$$[14] \qquad \rightarrow P_{right}(x) = \frac{-1000x^2 + 9646x - 22803}{352}$$

I then graphed [12], [13], and [14] in Desmos to model the egg:

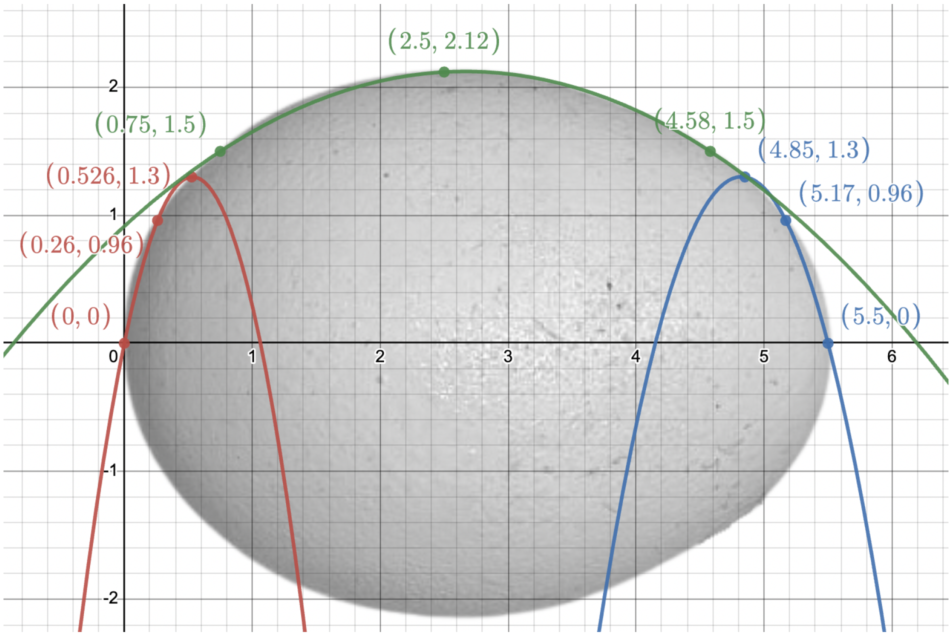

Figure 2.10: Three quadratic functions modeling three different sections of the egg

Now that one-half of the egg is modeled, I am going to reflect the functions by the x-axis by writing their relations and then plotting it on Desmos:

$$[15] \qquad f(x) = \begin{cases} \pm(-4.589567x^2 + 4.885595x) & 0 \leq x \leq 0.526 \\ \pm(-0.170330x^2 + 0.907857x + 0.914918) & 0.526 \leq x \leq 4.85 \\ \pm(-2.840909x^2 + 27.403409x - 64.78125) & 4.85 \leq x \leq 5.5 \end{cases}$$

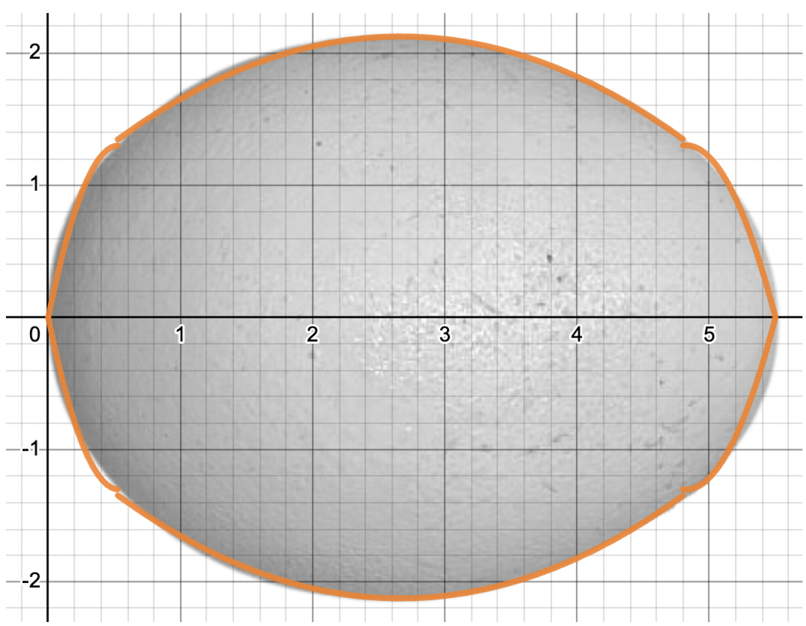

Figure 2.11: Graphical Presentation of [15]

As shown in **Figure 2.11**, this model is very precise and close to the perfect model. This can perhaps be because of the Lagrange formula which was used to model the egg. Using polynomials to model half an egg and reflect them is a very famous method used, however, Lagrange contributes to this method with more precision. However, some curvatures don't lay on the shape perfectly, Moreover, although the shape has shown to be overall symmetrical, at some points it is not perfect. Although the modeling above is completely acceptable, to increase the accuracy of the model two changes can be applied to the method. Firstly, the Lagrange formula could've been used to examine both halves of the egg, which would solve the problem of the egg not being 'perfectly' symmetrical. Secondly, I could've used a higher degree of polynomials, resulting in higher i and n, which would therefore make the equation and calculations far more complex while resulting in higher precision.

2.4 Juxtaposition

The main difference between the three methods used to model the shape of a chicken egg was precision and generality. The shape executed from the third method was for sure the most precise, as it was in regards to the shape's coordinates and measures specifically. The first method also looked at the shape of the specific egg, although minimally compared to the third method. The second method was the most general model, as it only looked at the length of the egg, and even with more than ±0.1cm uncertainty, the second method would still give a very similar output to what we had. As we are going to use the model to calculate the volume and surface area of the chicken egg, it is important to choose a model which has a balance between generality and precision. Therefore the following calculation in this exploration will be done in regard to the ellipse equation method of modeling. The method was chosen as it has a simpler formula than the oviform curve's formula, and is smoother and more general than the Lagrange interpolation method, as can be seen in the comparison of **Figure 2.11** and **Figure 2.4**. All in all, each method has its own pros and cons, and its usability is dependent on the application.

3. Finding the Volume

To find the volume of the egg, I am using the ellipse equation, [6], which was discussed before. As we are finding the volume of the egg, I started by extracting x from [6] and making it subject:

$$\frac{x^2}{4.4} + \frac{y^2}{7.5 + (0.5)y} = 1 \rightarrow x^2 \cdot (7.5) + (0.5)yx^2 + (4.4)y^2 = (7.5) \cdot (4.4) + (4.4) \cdot (0.5)y$$

$$[16] \qquad \rightarrow x = \sqrt{\frac{(-4.4)y^2 + (4.4)(0.5)y + (4.4)(7.5)}{(7.5) + (0.5)y}} = \sqrt{4.4 \cdot (1 - \frac{y^2}{7.5 + (0.5)y})}$$

According to the Math AA HL formula booklet, the volume is calculated by evaluating the integral between two desired limits of a function, rotated 90° (2π) about the y-axis (CliffsNotes):

$$[17] \qquad V = \pi \int_a^b [f(y)]^2 dy$$

The limits of the integral will be the y-intercepts in **Figure 2.4**, therefore $a = -2.5$, and $b = 3$. By substituting [16] with $f(y)$ in [17] we will have:

$$[18] \qquad V = 4.4\pi \int_{-2.5}^{3} 1 - \frac{y^2}{7.5 + 0.5y} dy$$

After applying the rules of integration and simplifying the expression we get:

$$V = \frac{22\pi}{5} \cdot (\frac{671}{4} + 900 \ln(\frac{5}{6})) = \frac{22\pi}{5} \cdot \frac{671 + 3600 \ln(\frac{5}{6})}{4} = \frac{11\pi}{5} \cdot \frac{671 + 3600 \ln(\frac{5}{6})}{2}$$

$$[19] \qquad \rightarrow V = \frac{11\pi \cdot (671 + 3600 \ln(\frac{5}{6})}{10} = \frac{7381\pi + 39600\pi \cdot \ln(\frac{5}{6})}{10}$$

Using Wolfram Alpha, the result of [19] and therefore the approximate volume of the egg is:

$$V \approx 50.6\ cm^3$$

4. Finding the Surface Area

The formula used to calculate the surface area of an egg according to Mathematica (Shuhao Cao) is:

$$A = 2\pi \int_a^b y\sqrt{1 + (\frac{dy}{dx})^2}dx$$

However, to make this formula easier to use, I decided to translate the derivative to $f(y)$, and therefore had this as the result:

$$[20] \qquad A = 2\pi \int_a^b f(y)\sqrt{1 + [f'(y)]^2}dy$$

Before substituting values in [20], I decided to rewrite the equation for x found in [16] to make the process simpler:

$$[21] \qquad x = \sqrt{4.4 \cdot (1 - \frac{y^2}{7.5 + (0.5)y})} = 2.1\sqrt{1 - \frac{y^2}{7.5 + (0.5)y}}$$

I then computed the derivative and then simplified it:

$$\frac{dx}{dy} = 2.1 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1 - \frac{y^2}{7.5 + (0.5)y}}} \cdot \frac{2y(7.5 + 0.5y) - y^2}{(7.5 + 0.5y)^2} = \frac{21}{2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1 - \frac{y^2}{7.5 + 0.5y}}} \cdot \frac{3y}{(\frac{15 + y}{2})^2}$$

$$21 \cdot \frac{1}{\sqrt{1 - \frac{y^2}{7.5 + (0.5)y}}} \cdot \frac{3y}{(15 + y)^2} = \frac{63}{\sqrt{1 - \frac{y^2}{7.5 + (0.5)y}}(15 + y)^2} = \frac{63}{\frac{\sqrt{15 + y - 2y^2}(15 + y)^2}{\sqrt{15 + y}}}$$

$$[22] \qquad = \frac{63y\sqrt{15 + y}}{\sqrt{-2y^2 + y + 15 \cdot (15 + y)^2}}$$

Now I finally have all the values required to find the surface area of the egg. I will be doing so by substituting [21] with $f(y)$ and [22] with $f'(y)$ in [20], which will result in:

$$[23] \qquad A = 4.2\pi \int_{-2.5}^{3} \sqrt{1 - \frac{y^2}{7.5 + (0.5)y}} \cdot \sqrt{1 + \frac{63y\sqrt{15+y}}{\sqrt{-2y^2 + y + 15} \cdot (15+y)^2}^{2}}\, dy$$

I then inserted [23] into Wolfram Alpha to find A, and therefore the approximate value for the surface area of the egg is:

$$A \approx 68.1\ cm^2$$

5. Conclusion

5.1 Critical Analysis

To bring this investigation to a conclusion, I originally wanted to use the volume of revolution function in Python's math library to compare answers and evaluate the accuracy of the investigation. However, the method, although calculations being done by computer, would not have been accurate. Firstly, the modeling would've been done by the computer with an oviform curve approach which would have not been the most accurate. Secondly, there is no function available for calculating the surface area of an egg, and the function should've been written by myself which would've been over-complicated for the aim of this investigation.

As in search of other ways to calculate the volume and surface of an egg, I reached a fascinating paper written by **Nobuo Yamamoto**, which seemed highly accurate in modeling and calculations. The approaches taken in Yamamoto's paper are beyond both the scope of this investigation and my own knowledge and understanding, and therefore will not be discussed, and only the equations will be used to validate my answers.

According to Yamamoto, the best way to model an egg is with the following formula (Nobuo Yamamoto):

$$[24] \quad (x^2+y^2)^2 = ax^2 + (a-b)xy^2\ ,\ a \geq b \geq 0$$

To find a and b, I used Desmos and plotted [24] with values that would model my egg, as accurately as possible. And therefore found $a = 5.5$ and $b = 3$.

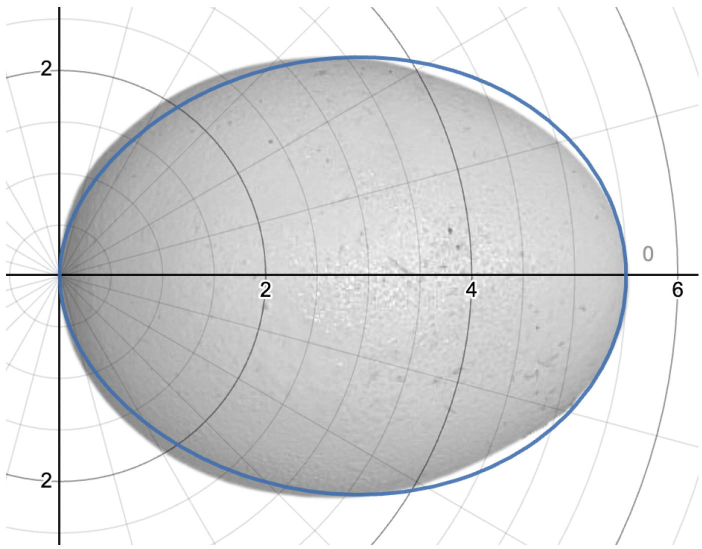

Figure 2.12: presentation of [24] with a = 5.5 and b = 3

Note that although the modeling does not seem the most accurate, the amount of white spaces is close to equal to the parts of the egg left out, and therefore the model is very accurate for this investigation's purpose.

According to Yamamoto, the volume of revolution using this model is calculated with:

$$R = \frac{\pi}{2}(\frac{a}{6b}(a+b)^3 + \frac{1}{60b^2}((a-b)^5 - (a+b)^5) - \frac{1}{6}a^3 - \frac{1}{2}a^2b)$$

$$\rightarrow R = \frac{\pi}{2}(\frac{5.5}{6(3)}(5.5+3)^3 + \frac{1}{60(3)^2}((5.5-3)^5 - (5.5+3)^5) - \frac{1}{6}(5.5)^3 - \frac{1}{2}(5.5)^2(3))$$

$$[25] \quad \rightarrow R = 49.3\ cm^3$$

The calculations are reassuring as my original answer (50.6_{cm^3}) is very close to [25]. Furthermore, the surface area can, according to Yamamoto, be calculated with:

$$[26] \qquad S = 2\pi\int_0^a |y|\sqrt{1+(\frac{dy}{dx})^2}dx$$

The values of $+y$ and $\frac{dy}{dx}$ are given in the paper, which are:

$$[27] \qquad y = +\frac{\sqrt{(a-b)-2x+\sqrt{(a-b)^2}}}{\sqrt{2}}\sqrt{x} \rightarrow y = \frac{\sqrt{(5.5-3)-2x+\sqrt{(5.5-3)^2}}}{\sqrt{2}}\sqrt{x}$$

$$\frac{dy}{dx} = \pm\frac{1}{2}(\sqrt{\frac{(a-b)-2x+\sqrt{4bx+(a-b)^2}}{2x}} + (\frac{b}{\sqrt{4bx+(a-b)^2}} - 1)\cdot\sqrt{\frac{2x}{(a-b)-2x+\sqrt{4bx+(a-b)^2}}})$$

[28]

$$\rightarrow \frac{dy}{dx} = \frac{1}{2}(\sqrt{\frac{(5.5-3)-2x+\sqrt{4(3)x+(5.5-3)^2}}{2x}} + (\frac{3}{\sqrt{4(3)x+(5.5-3)^2}} - 1)\cdot\sqrt{\frac{2x}{(5.5-3)-2x+\sqrt{4(3)x+(5.5-3)^2}}})$$

Substituting [27] with $|y|$ and [28] with $\frac{dy}{dx}$ in [26] will result in:

$$S = 2\pi\sqrt{x}\int_0^a \frac{\sqrt{(5.5-3)-2x+\sqrt{(5.5-3)^2}}}{\sqrt{2}}\sqrt{1+(\frac{1}{2}(\sqrt{\frac{(5.5-3)-2x+\sqrt{4(3)x+(5.5-3)^2}}{2x}} + (\frac{3}{\sqrt{4(3)x+(5.5-3)^2}} - 1)\cdot\sqrt{\frac{2x}{(5.5-3)-2x+\sqrt{4(3)x+(5.5-3)^2}}}))^2}dx$$

$$[29] \qquad \rightarrow S = 66.7cm^2$$

Similarly again, the value of [29] is very close to my original value for the Surface of the egg.

5.2 Verdict

According to the experiment which took place in the previous section, we can conclude that my answers had an uncertainty/error of approximately 1.02%, which indicates the high accuracy of the investigation. Therefore as I am satisfied with my results, to conclude this investigation, the best method to model an egg is using the Ellipse equation, furthermore, the volume and surface area of the specific chicken egg first shown in **Figure 2.4**, are 50.6 and 68.1 respectively. Although the investigation was shown to be overall precise and accurate, there were uncertainties throughout. Some ways of minimizing them are to perhaps compare each modeling method furthermore by performing calculations on precise points of the egg, by taking more modeling approaches, or even using computer algorithms for every single calculation, and using higher significant figures in calculations.

Overall, the investigation was very interesting for me, and the time I spent on it was joyful as it allowed me to discover various new relations and interesting mathematical connections within an everyday object I see every day. This investigation may not be very impactful, however, it made me learn new mathematical theories, use new methods, and increase my mathematical knowledge and understanding. Lastly, the perhaps boldest impact of this investigation, is that I will never look at a chicken egg the same way again.

6. Bibliography

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7. Appendices

Appendix A

A.1: Wolfram Algorithm used to find the axis of symmetry

Note: The code is from source [2]

```
image = import["sepehrak/documents/IB/math_IA/egg_bg.png"];

image = FillingTransform[Closing[image, 2]]

comp = ComponentMeasurement [
    image, {"Centroid", "Orientation", "Length"}]

HighlightImage[img,
 comp /. (index_ -> {centroid_, orientation_, length_}) :> {
    Rotate[Line[{centroid - {length/2, 0}, centroid + {length/2,
0}}],
      orientation]
    }]
```

Appendix B

B.1: Python code used to plot figure 2.6

```
import numpy as np
import matplotlib.pyplot as plt

def xpart(x, L, b, w):
   numerator = L ** 2 - 4 * (x ** 2)
   denom = L**2 + 8*w*x + 4*w**2
   return (b**2/4)*(numerator/denom)

x = np.linspace(-3, 3, num=50)
y = np.linspace(-3, 3, num=50)
x, y = np.meshgrid(x, y)

G1 = xpart(x, L=5.5, b=4.2, w=0)
G2 = xpart(x, L=5.5, b=4.2, w=0)

Y = y**2
```

```
fig = plt.figure(figsize=(10, 4))

fig.add_subplot(121)
plt.contourf(Y-G1, cmap='magma')
plt.title(r'$L=5.5, B=4.2, W=0$', fontsize=12)
fig.add_subplot(122)
plt.title(r'$L=5.5, B=4.2, W=0$', fontsize=12)
plt.contourf(Y-G2, cmap='magma')
plt.tight_layout()

plt.show()
```

B.2: Python code used to plot figure 2.7

```
import numpy as np
import matplotlib.pyplot as plt

def xpart(x, L, b, w):
   numerator = L ** 2 - 4 * (x ** 2)
   denom = L**2 + 8*w*x + 4*w**2
   return (b**2/4)*(numerator/denom)

x = np.linspace(-3, 3, num=50)
y = np.linspace(-3, 3, num=50)
x, y = np.meshgrid(x, y)

G1 = xpart(x, L=5.5, b=4.2, w=0)
G2 = xpart(x, L=5.5, b=4.2, w=0.65)

Y = y**2

fig = plt.figure(figsize=(10, 4))

fig.add_subplot(121)
plt.contourf(Y-G1, cmap='magma')
plt.title(r'$L=5.5, B=4.2, W=0$', fontsize=12)
fig.add_subplot(122)
plt.title(r'$L=5.5, B=4.2, W=0.65$', fontsize=12)
plt.contourf(Y-G2, cmap='magma')
plt.tight_layout()

plt.show()
```